

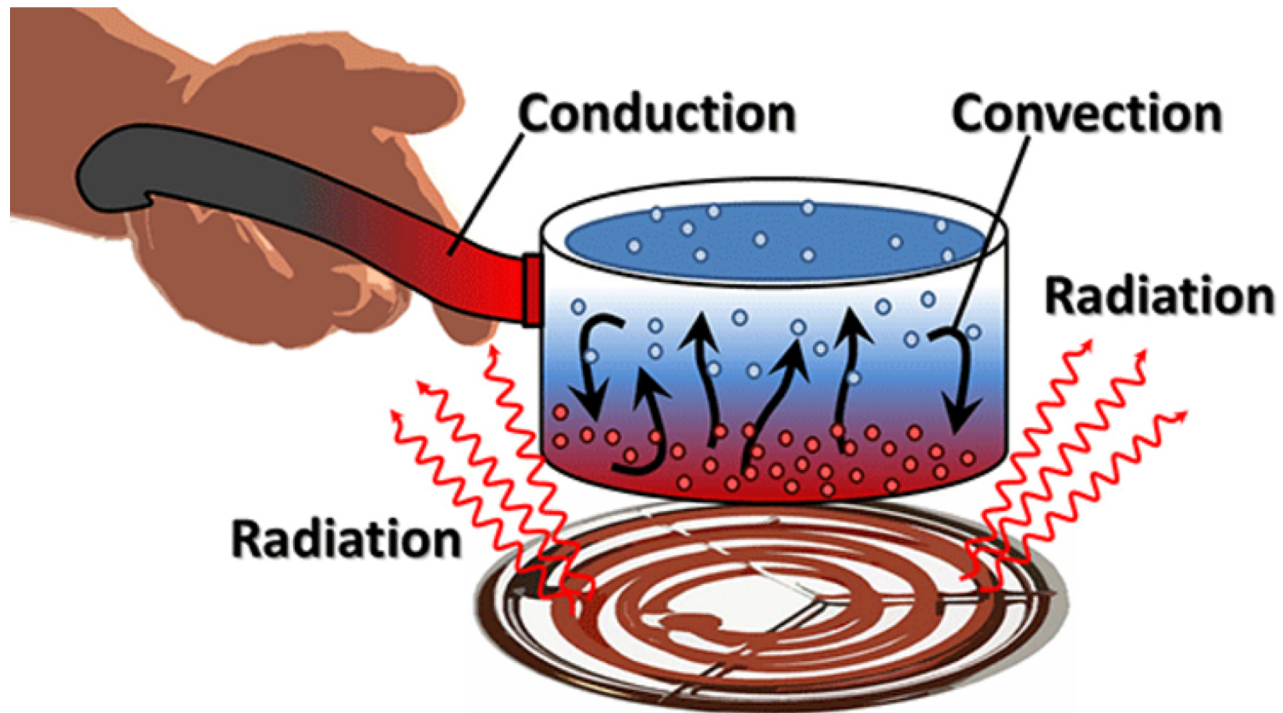
# Strahlungsantrieb von Treibhausgasen: viel Lärm um fast nichts

A Talk at the 14<sup>th</sup> International EIKE-Klima-und  
Energiekonferenz am 12.-13. November  
2021 in Gera, Germany

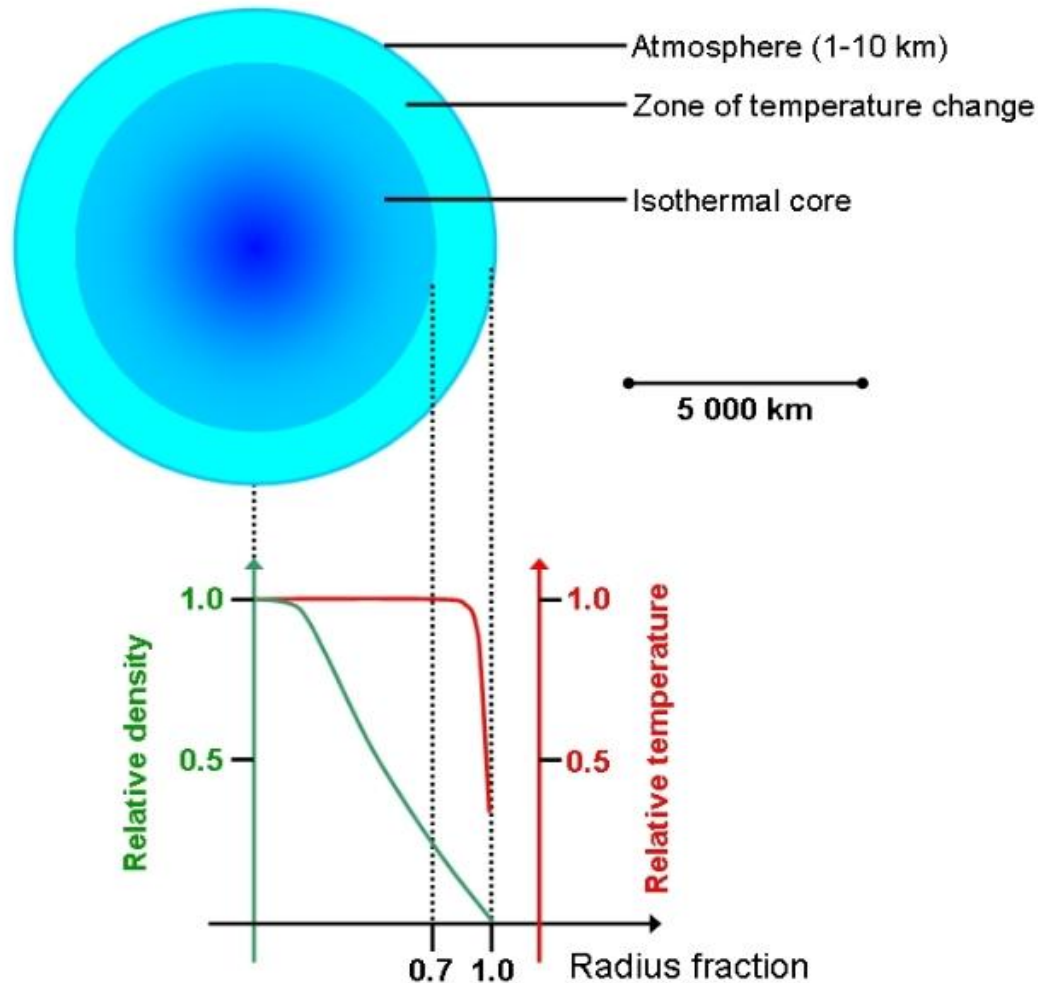
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## HEAT TRANSFER MECHANISMS



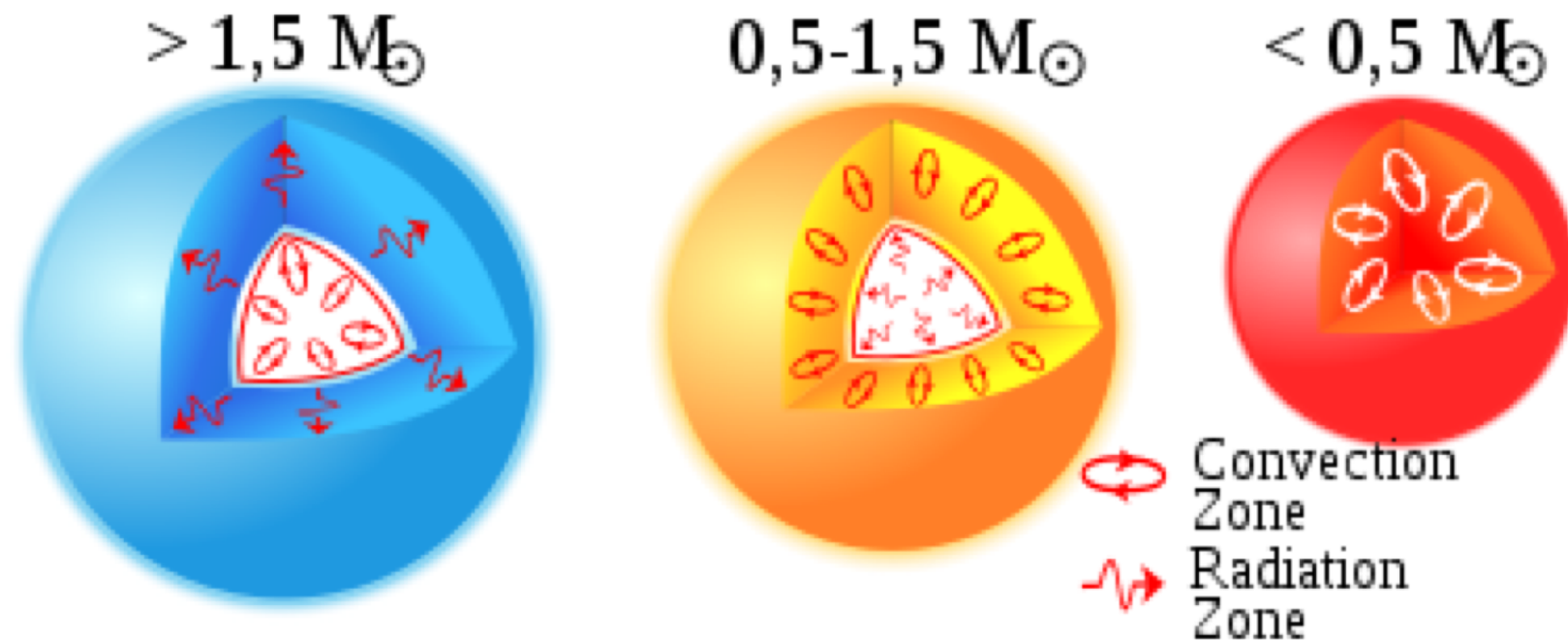
### Structure of a White Dwarf



Conductive heat transfer over distances like Earth's atmosphere or within stars is not normally important compared to radiative or convective heat transfer. But conductive heat transfer by a degenerate electron gas does dominate in white dwarf stars and it makes the interior almost isothermal.

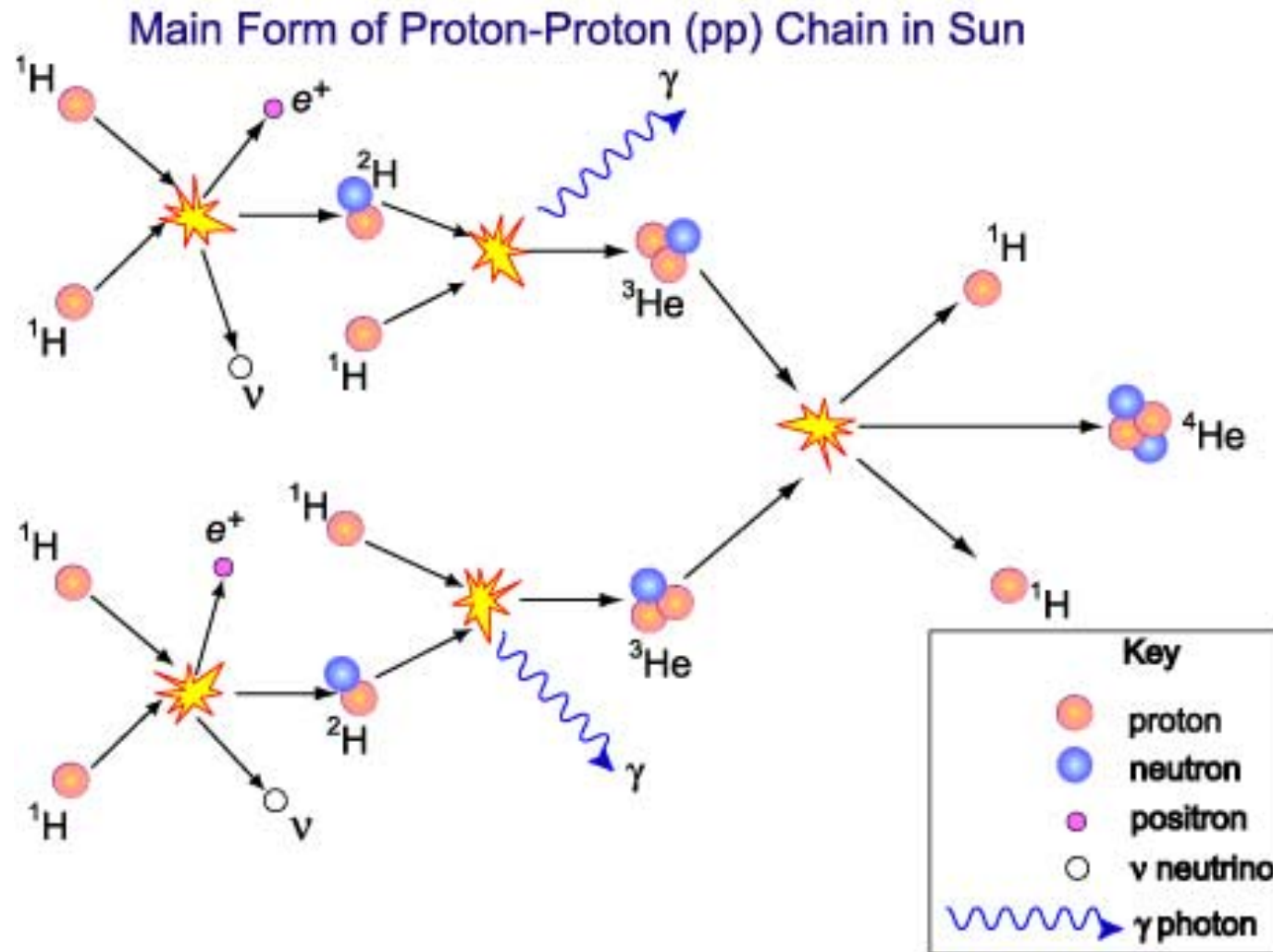
<https://cronodon.com/SpaceTech/WhiteDwarf.html>

# Heat Transfer of Stars



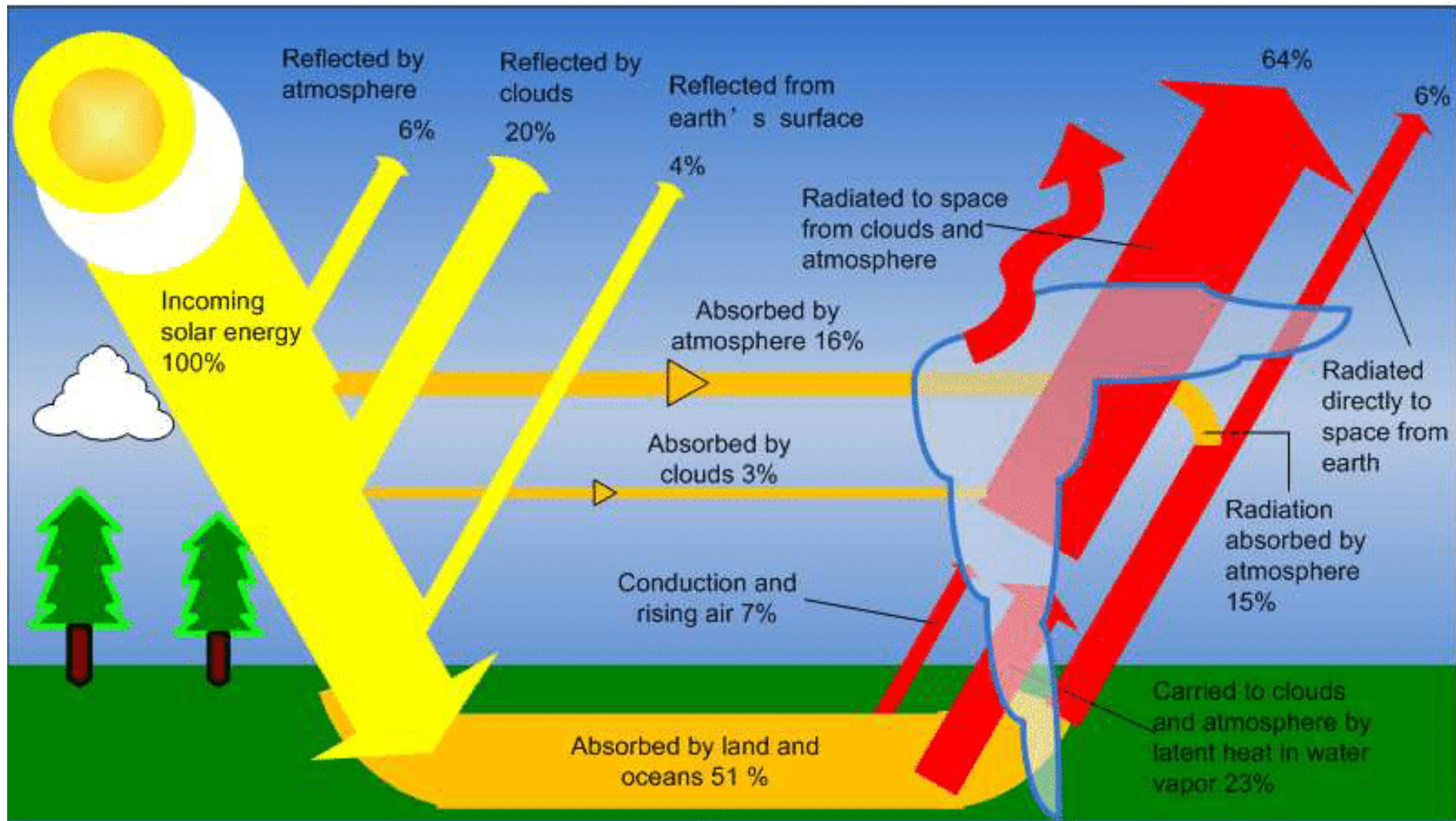
[https://en.wikipedia.org/wiki/Stellar\\_structure](https://en.wikipedia.org/wiki/Stellar_structure)

# Heat Source in Sun's Core is Nuclear Fusion

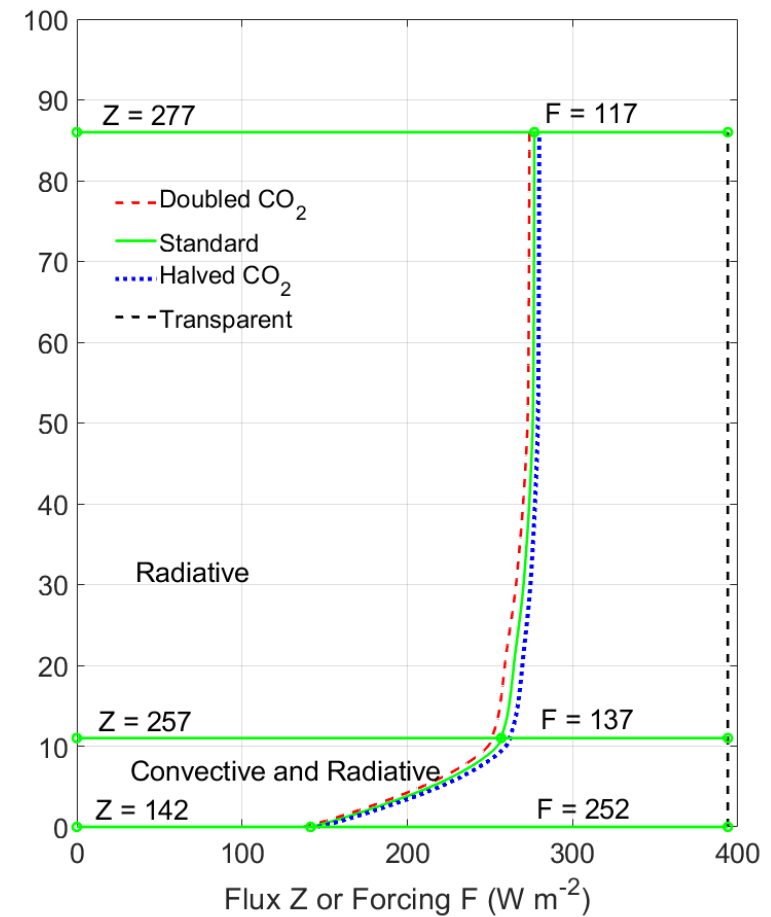
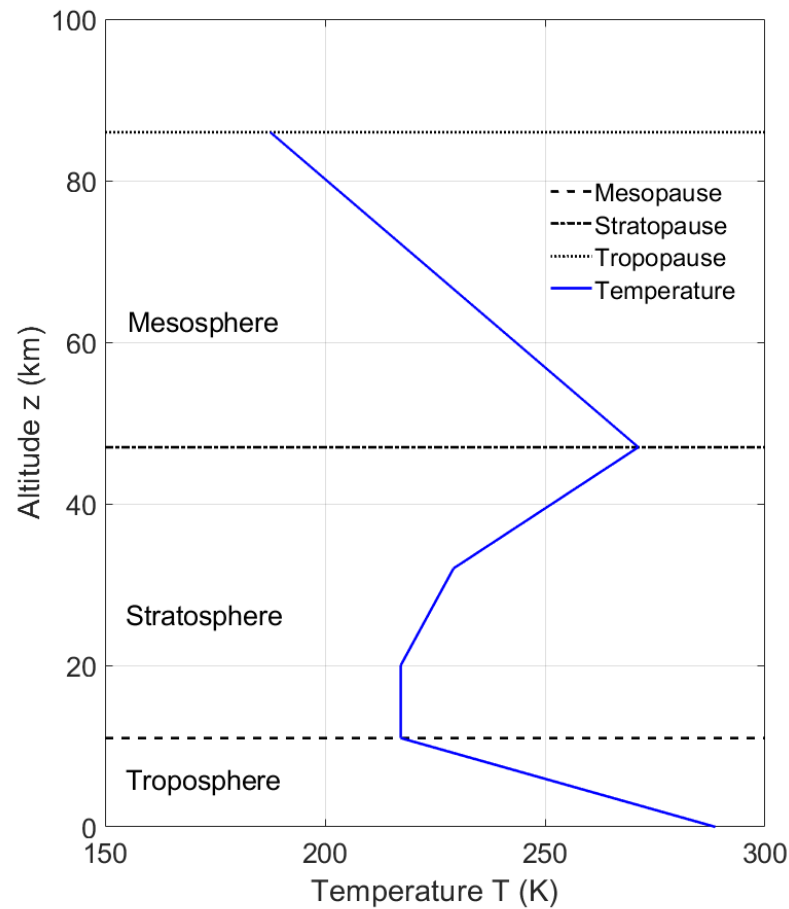




Heat source for Earth's surface is almost all sunlight.



# Upward Radiative Flux $Z(z)$ Versus Altitude $z$



<https://arxiv.org/pdf/2006.03098.pdf>

Central temperature of heated object is controlled by central heating rate and by how efficiently heat is “shed” to surroundings

**The Sun**

$$T_h - T_c = 15,000,000 \text{ C}$$



277 W/m<sup>3</sup>

**A compost pile**

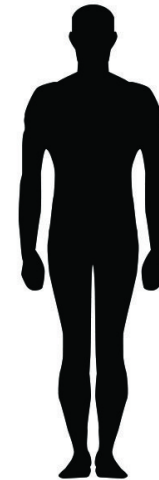
$$T_h - T_c = 40 \text{ C}$$



Maximum is about the same as the Sun's core.

**A human being**

$$T_h - T_c = 20 \text{ C}$$



About 1000 W/m<sup>3</sup> at rest



- For a fixed solar heating rate,  $\dot{Q}$ , of Earth's surface the difference between the “hot” surface temperature  $T_h$  of about 300 K, and cold outer space temperature  $T_c$  of about 2.73 K is proportional to that thermal resistance  $R$  of the atmosphere.
- One of many factors (clouds, convection, variation of temperature with altitude, etc.) that influence  $R$  is the concentration of greenhouse gases

$$R\dot{Q} = T_h - T_c \quad \text{Newton's law of cooling}$$

$$RI = V_h - V_c \quad \text{Ohm's law}$$

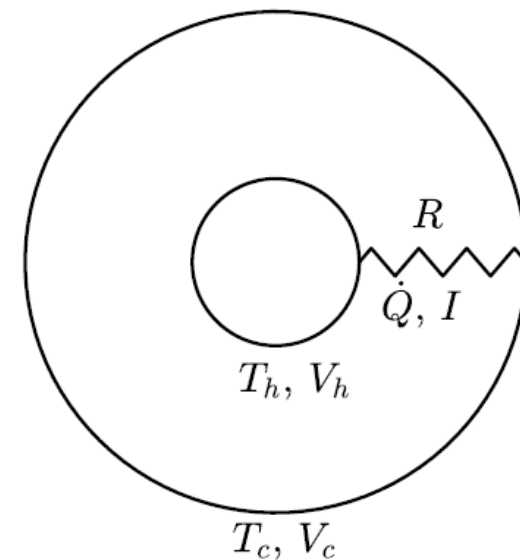
$R$  = Thermal or electrical resistance

$\dot{Q}$  = Heat current

$I$  = Electrical current

$T_h, T_c$  = Hot and cold temperatures

$V_h, V_c$  = “Hot and cold” voltages



Fundamental evolution equations of physics:

$$\mathbf{F} = m\mathbf{a},$$

Classical mechanics

$$i\hbar\frac{\partial}{\partial t}\psi = H\psi,$$

Quantum mechanics

$$\nabla \cdot \mathbf{E} = 4\pi\rho,$$

Electrostatics

$$? = ?$$

Radiation transfer

# Equation of Transfer

$$\frac{\partial}{\partial z} I(z, \mu) = \frac{\alpha}{\mu} \left[ -I(z, \mu) + (1 - \tilde{\omega}) B(z) + \frac{\tilde{\omega}}{2} \int_{-1}^1 p(\mu, \mu') I(z, \mu') d\mu' \right]$$

$I(z, \mu)$  = Intensity (radiance), the “state function” of radiation

$z$  = Altitude of intensity

$\mu$  = Direction cosine of intensity

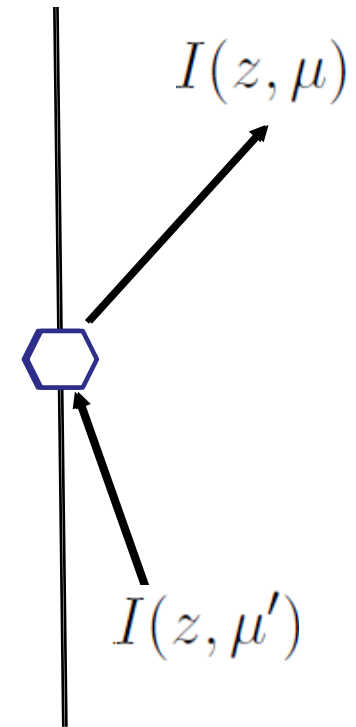
$\alpha$  = Extinction coefficient from absorption and scattering

$\tilde{\omega}$  = Single-scattering albedo

$B(z)$  = Planck intensity =  $\frac{2hc^2\nu^3}{e^{\nu c h/(kT(z))} - 1}$

$T(z)$  = Absolute temperature at altitude  $z$

$p(\mu, \mu')$  = Single-scattering phase function

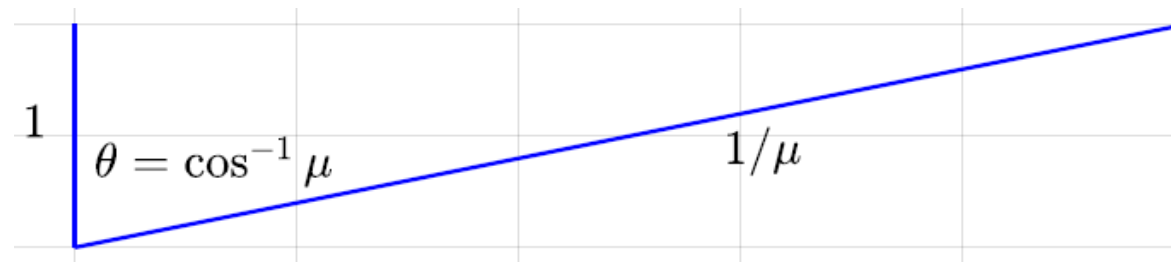


A great historical review of the equation of transfer can be found here:

[https://www.oceanopticsbook.info/packages/iws\\_l2h/conversion/files/Mobley\\_EvolutionRTT\\_draft.pdf](https://www.oceanopticsbook.info/packages/iws_l2h/conversion/files/Mobley_EvolutionRTT_draft.pdf)

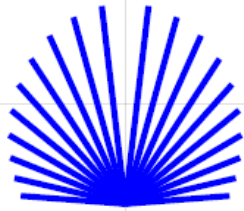
For cloud-free regions of Earth's atmosphere, there is negligible scattering of thermal radiation. Greenhouse molecules only absorb and emit thermal radiation. Their single-scattering albedo is negligibly small ( $\tilde{\omega} = 0$ ). The equation of transfer for the cloud-free atmosphere therefore reduces to the Schwarzschild Equation:

$$\frac{\partial}{\partial z} I(z, \mu) = \frac{\alpha}{\mu} [-I(z, \mu) + B(z)]$$

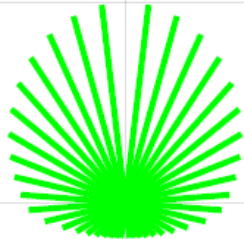


Slant path longer than vertical path by factor  $1/\mu$

## Intensity versus direction for various altitudes for frequencies where the atmosphere is optically thick



Above the cold, radiative top of atmosphere.  
No downward radiation, limb darkening.



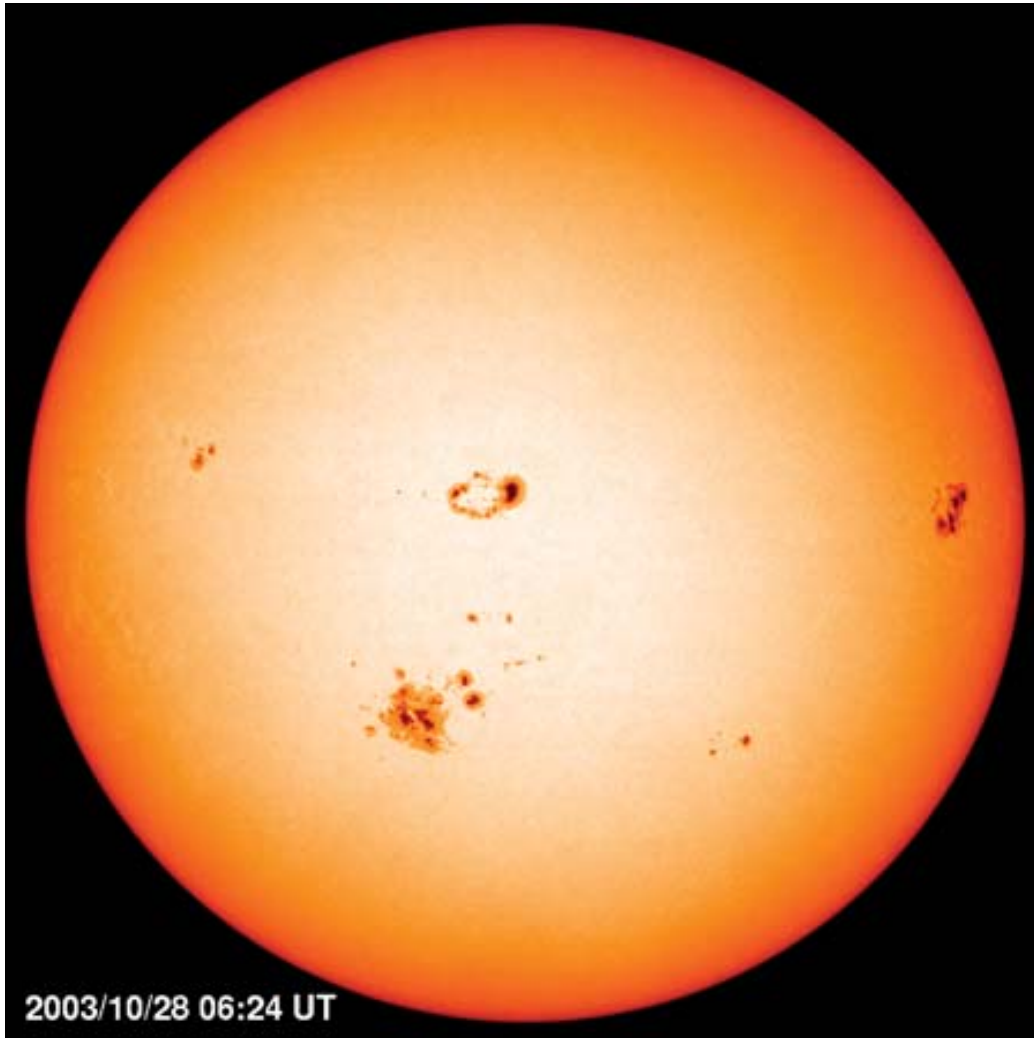
Just below the cold, radiative top of atmosphere.  
More upward than downward radiation



Warm, near ground level, isotropic,  
negligible heat transport

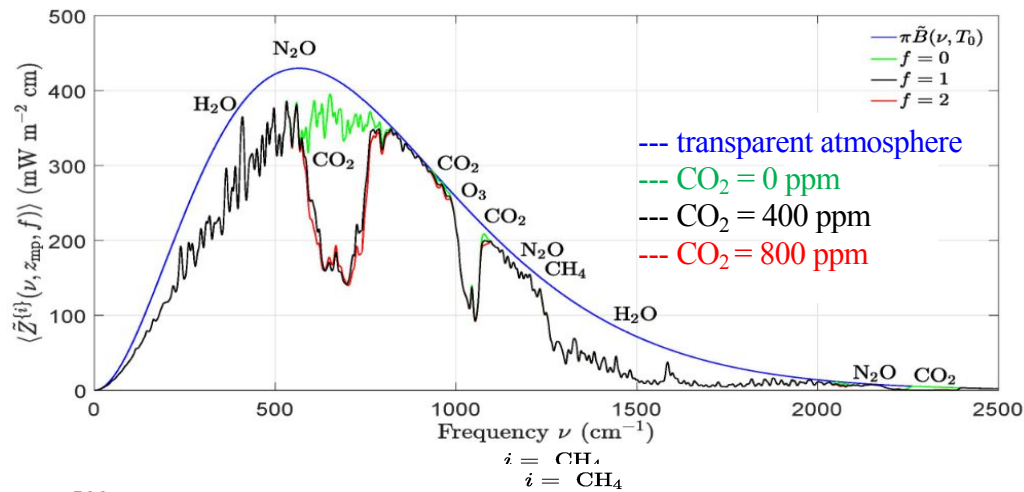


# Limb darkening of the Sun

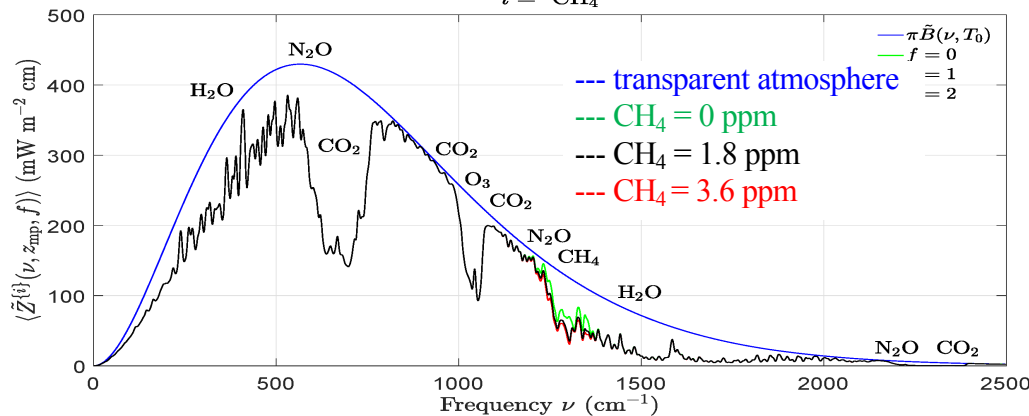


Photosphere of the Sun with limb darkening. Image taken by the Solar and Heliospheric Observatory satellite, Oct. 29, 2003.

# Greenhouse gases slow the cooling of Earth by thermal radiation to space.



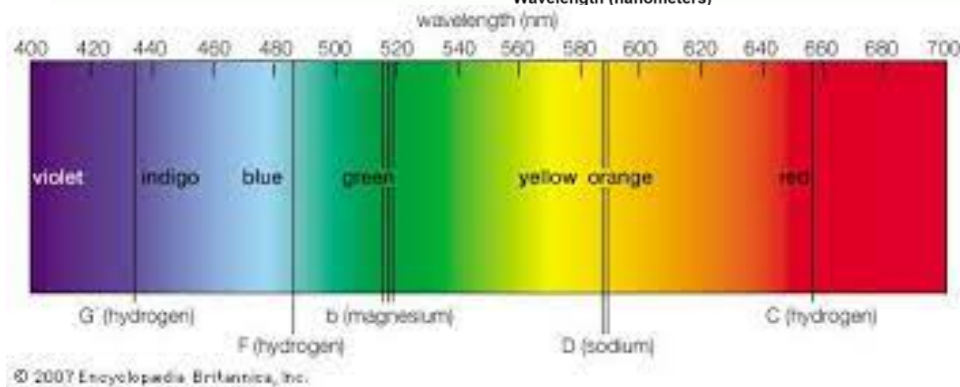
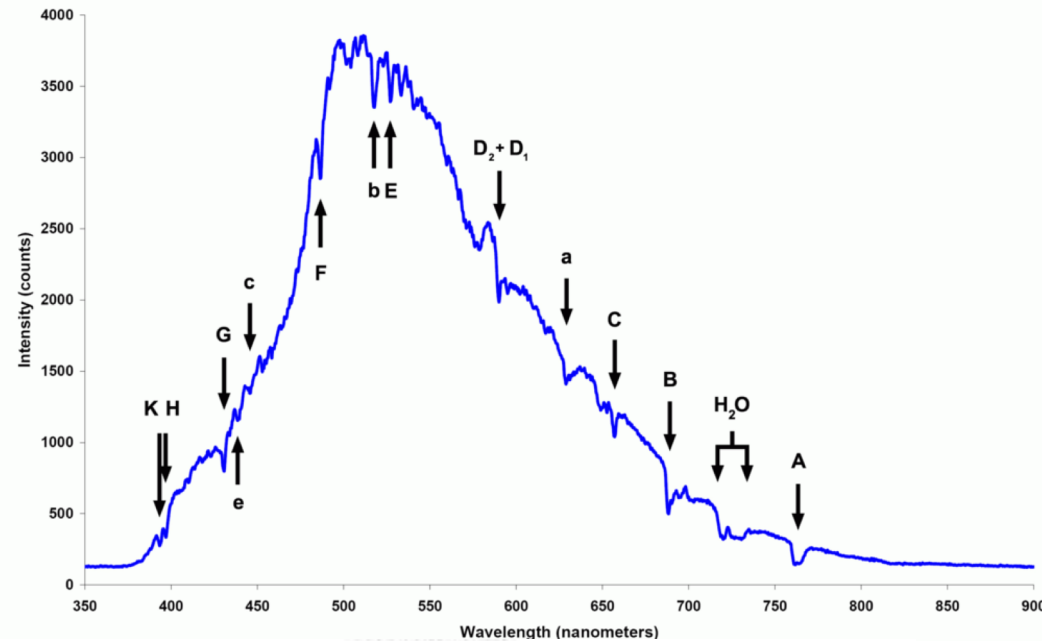
**Max Planck**  
1858-1947



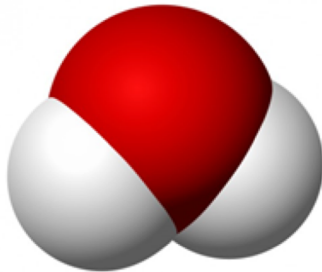
**Karl Schwarzschild**  
1873-1916

Greenhouse-gases modify the emission of Earth's atmosphere in the same way as atoms and ions in the atmosphere of the Sun produce Fraunhofer dark lines.

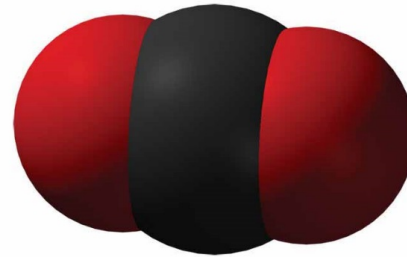
Joseph von Fraunhofer  
1787 - 1826



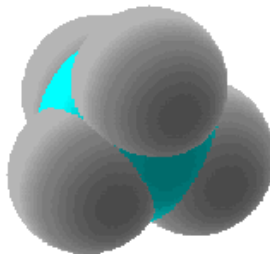
# Greenhouse Gas Molecules



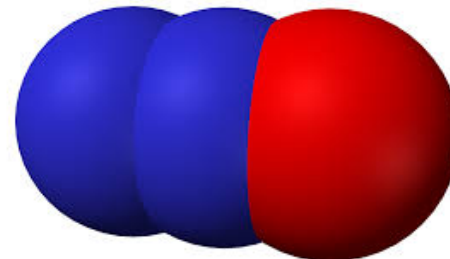
Water Vapor  
(Major)



Carbon Dioxide  
(Minor)

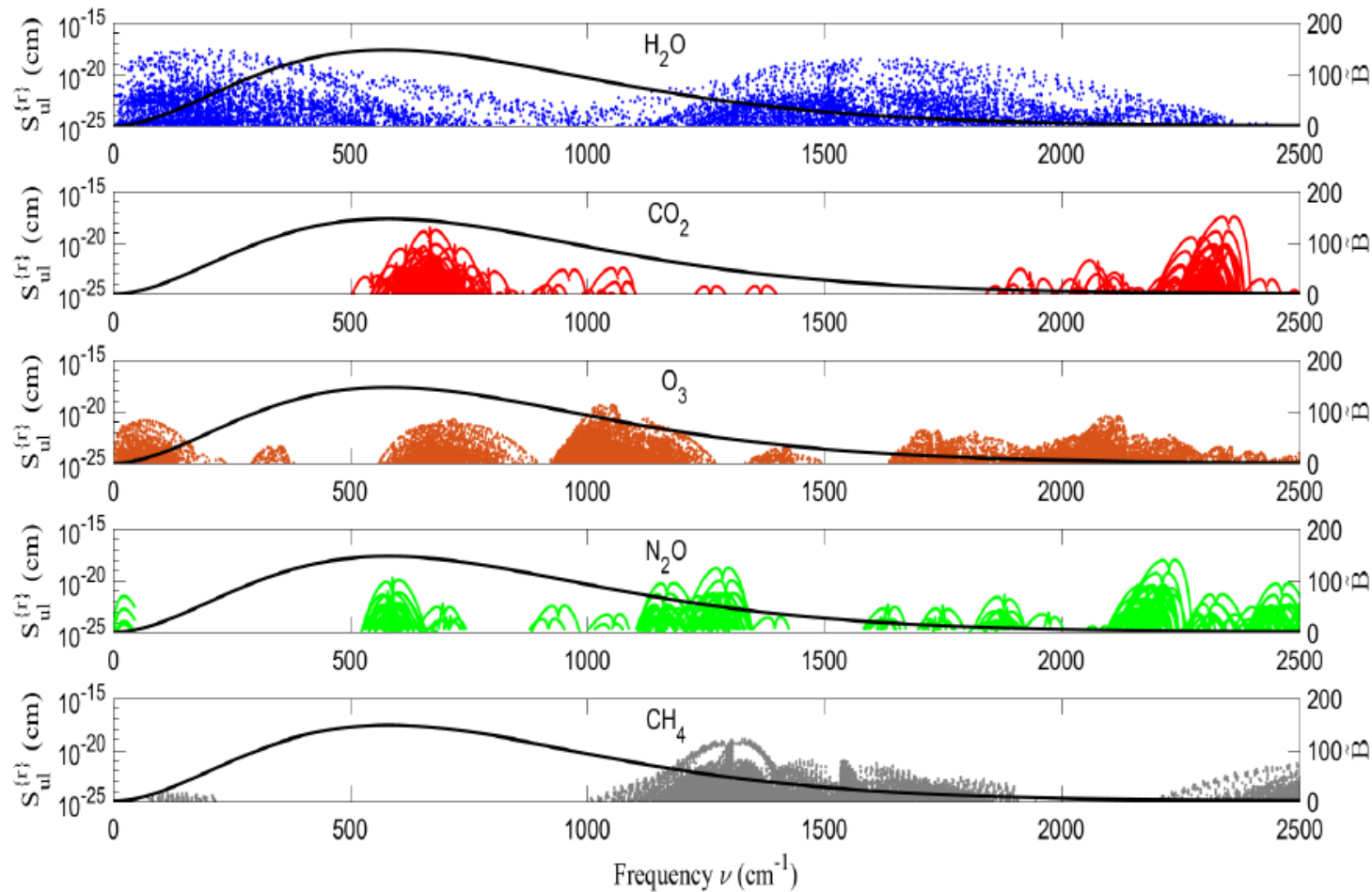


Methane  
(Very Minor)

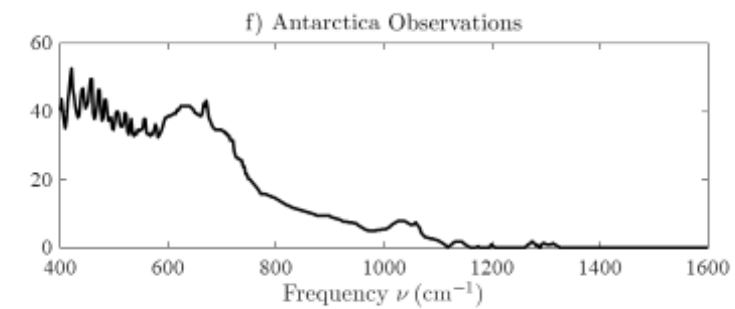
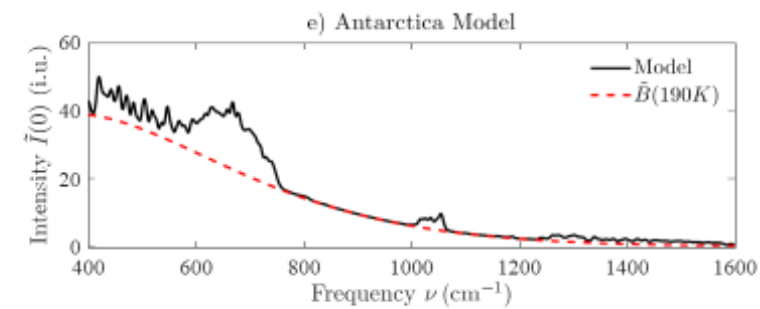
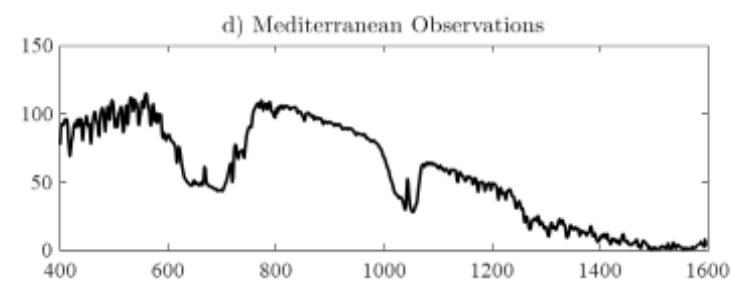
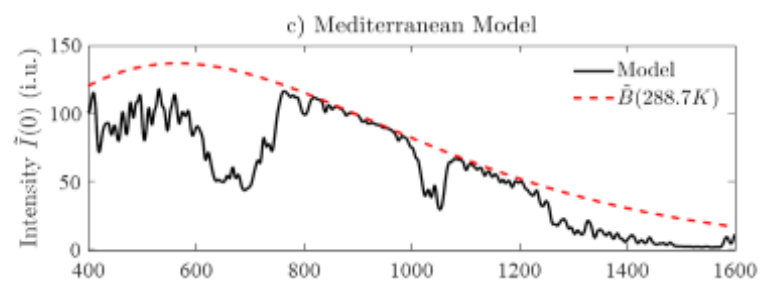
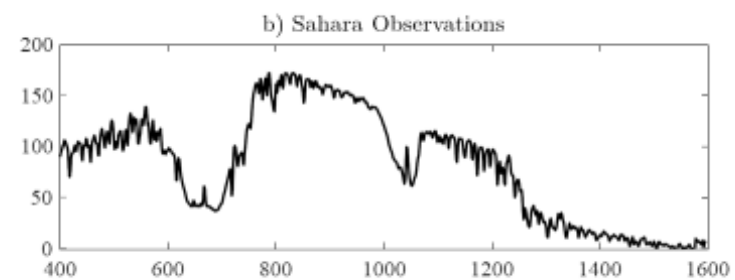
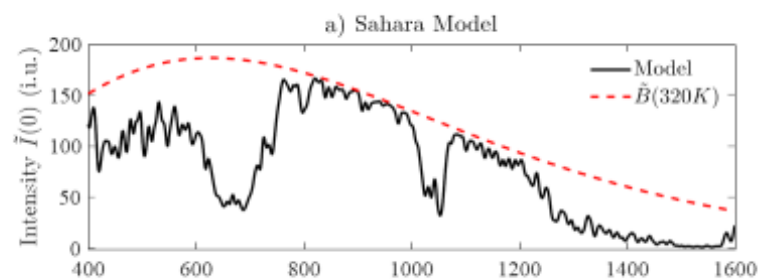


Nitrous Oxide  
(Very Minor)

The Sun's Planck spectrum is modified by a few dozen lines of atoms and atomic ions. The Earth's Planck spectrum is modified by hundreds of thousands of molecular vibration-rotation lines.







## Zeroth-order estimate of warming needed to compensate for attenuation of flux to space by 2 x CO<sub>2</sub>

$$\frac{\Delta Z}{Z} \approx 1.1\%$$

Decrease of flux to space from 2 x CO<sub>2</sub>  
 Radiation transfer calculation for cloud-free skies

and

$$Z \approx \sigma T^4$$

Stefan-Boltzmann radiation law, (T<sup>3</sup> fits better than T<sup>4</sup> for atmosphere with greenhouse gases)

so

$$\frac{\Delta Z}{Z} \approx 4 \frac{\Delta T}{T}$$

or

$$\frac{\Delta T}{T} \approx \frac{1}{4} \times 1.1\%,$$

but

Mean surface temperature

$$T \approx 290 \text{ K},$$

so

$$\Delta T \approx 0.80 \text{ C.}$$

Zeroth-order estimate of warming from 2 x CO<sub>2</sub>